OCR
Oxford Cambridge and RSA

Wednesday 3 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

5089096931

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **24** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Vectors

1 The point A has coordinates (2, 5, 4) and the line BC has equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 25 \\ 43 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 15 \\ 25 \end{pmatrix}.$$

You are given that AB = AC = 15.

(i) Show that the coordinates of one of the points B and C are (4, 10, 18). Find the coordinates of the other point. These points are B and C respectively. **[6]**

(ii) Find the equation of the plane ABC in cartesian form. **[4]**

(iii) Show that the plane containing the line BC and perpendicular to the plane ABC has equation $5y - 3z + 4 = 0$. **[4]**

The point D has coordinates (1, 1, 3).

(iv) Show that $|\overrightarrow{BC} \times \overrightarrow{AD}| = \sqrt{7667}$ and hence find the shortest distance between the lines BC and AD. **[7]**

(v) Find the volume of the tetrahedron ABCD. **[3]**

Option 2: Multi-variable calculus

2 A surface has equation $z = 3x^2 - 12xy + 2y^3 + 60$.

(i) Show that the point A $(8, 4, -4)$ is a stationary point on the surface. Find the coordinates of the other stationary point, B, on this surface. **[5]**

(ii) A point P with coordinates $(8+h, 4+k, p)$ lies on the surface.

(A) Show that $p = -4 + 3(h-2k)^2 + 2k^2(6+k)$. **[3]**

(B) Deduce that the stationary point A is a local minimum. **[3]**

(C) By considering sections of the surface near to B in each of the planes $x = 0$ and $y = 0$, investigate the nature of the stationary point B. **[4]**

(iii) The point Q with coordinates (1, 1, 53) lies on the surface.

Show that the equation of the tangent plane at Q is

$$6x + 6y + z = 65.$$ **[4]**

(iv) The tangent plane at the point R has equation $6x + 6y + z = \lambda$ where $\lambda \neq 65$.

Find the coordinates of R. **[5]**

Option 3: Differential geometry

3 Fig. 3 shows an ellipse with parametric equations $x = a\cos\theta$, $y = b\sin\theta$, for $0 \leqslant \theta \leqslant 2\pi$, where $0 < b \leqslant a$.
The curve meets the positive x-axis at A and the positive y-axis at B.

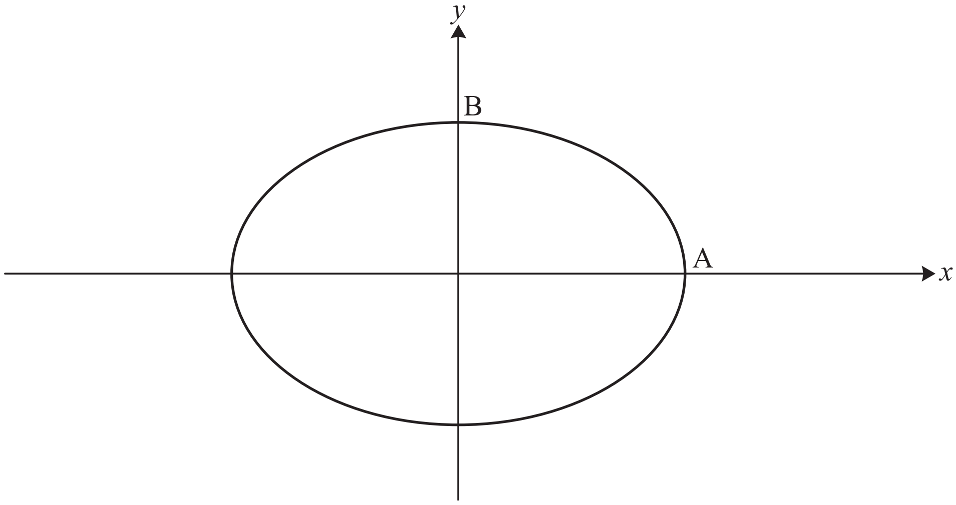

Fig. 3

(i) Show that the radius of curvature at A is $\dfrac{b^2}{a}$ and find the corresponding centre of curvature. **[7]**

(ii) Write down the radius of curvature and the centre of curvature at B. **[2]**

(iii) Find the relationship between a and b if the radius of curvature at B is equal to the radius of curvature at A. What does this mean geometrically? **[1]**

(iv) Show that the arc length from A to B can be expressed as

$$b\int_0^{\frac{\pi}{2}} \sqrt{1+\lambda^2 \sin^2\theta}\, d\theta,$$

where λ^2 is to be determined in terms of a and b.

Evaluate this integral in the case $a = b$ and comment on your answer. **[7]**

(v) Find the cartesian equation of the evolute of the ellipse. **[7]**

Option 4: Groups

4 M is the set of all 2×2 matrices m(a,b) where a and b are rational numbers and

$$\mathrm{m}(a,b)=\begin{pmatrix} a & b \\ 0 & \frac{1}{a} \end{pmatrix},\ a \neq 0.$$

(i) Show that under matrix multiplication M is a group. You may assume associativity of matrix multiplication. **[7]**

(ii) Determine whether the group is commutative. **[3]**

The set N_k consists of all 2×2 matrices m(k,b) where k is a fixed positive integer and b can take any integer value.

(iii) Prove that N_k is closed under matrix multiplication if and only if $k = 1$. **[4]**

Now consider the set P consisting of the matrices m(1,0), m(1,1), m(1,2) and m(1,3). The elements of P are combined using matrix multiplication but with arithmetic carried out modulo 4.

(iv) Show that $(\mathrm{m}(1,1))^2 = \mathrm{m}(1,2)$. **[2]**

(v) Construct the group combination table for P. **[4]**

The group R consists of the set $\{e, a, b, c\}$ combined under the operation *. The identity element is e, and elements a, b and c are such that

$a*a = b*b = c*c$ and $a*c = c*a = b$.

(vi) Determine whether R is isomorphic to P. **[4]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 An inspector has three factories, A, B, C, to check. He spends each day in one of the factories. He chooses the factory to visit on a particular day according to the following rules.

- If he is in A one day, then the next day he will never choose A but he is equally likely to choose B or C.
- If he is in B one day, then the next day he is equally likely to choose A, B or C.
- If he is in C one day, then the next day he will never choose A but he is equally likely to choose B or C.

(i) Write down the transition matrix, **P**. **[2]**

(ii) On Day 1 the inspector chooses A.

(*A*) Find the probability that he will choose A on Day 4. **[3]**

(*B*) Find the probability that the factory he chooses on Day 7 is the same factory that he chose on Day 2. **[4]**

(iii) Find the equilibrium probabilities and explain what they mean. **[4]**

The inspector is not satisfied with the number of times he visits A so he changes the rules as follows.

- If he is in A one day, then the next day he will choose A, B, C, with probabilities 0.8, 0.1, 0.1, respectively.
- If he is in B or C one day, then the probabilities for choosing the factory the next day remain as before.

(iv) Write down the new transition matrix, **Q**, and find the new equilibrium probabilities. **[3]**

(v) On a particular day, the inspector visits factory A. Find the expected number of consecutive further days on which he will visit factory A. **[3]**

Still not satisfied, the inspector changes the rules as follows.

- If he is in A one day, then the next day he will choose A, B, C, with probabilities 1, 0, 0, respectively.
- If he is in B or C one day, then the probabilities for choosing the factory the next day remain as before.

The new transition matrix is **R**.

(vi) On Day 15 he visits C. Find the first subsequent day for which the probability that he visits B is less than 0.1. **[3]**

(vii) Show that in this situation there is an absorbing state, explaining what this means. **[2]**

END OF QUESTION PAPER

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1 (i)	

1 (ii)	

1 (iii)	

1 (iv)

1 (v)

2 (i)	

2 (ii) (*A*)

2 (ii) (*B*)

2 (ii) (*C*)

2 (iii)

2 (iv)

3 (i)	

3 (ii)	
3 (iii)	

3 (iv)

3 (v)	

4 (i)	

4 (ii)	
4 (iii)	

4 (iv)	
4 (v)	

4 (vi)	

5 (i)	
5 (ii) (*A*)	

5 (ii) (*B*)	
5 (iii)	

5 (iv)	

5 (v)

5 (vi)	

5 (vii)	

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

1	**(i)**	Any point is $\begin{pmatrix}8\\25\\43\end{pmatrix}+\lambda\begin{pmatrix}4\\15\\25\end{pmatrix}$ $=((8+4\lambda),(25+15\lambda),(43+25\lambda))$ $\text{AB}=((8+4\lambda),(25+15\lambda),(43+25\lambda))-(2,5,4)$ $=((6+4\lambda),(20+15\lambda),(39+25\lambda))$ Distance AB $=\sqrt{(6+4\lambda)^2+(20+15\lambda)^2+(39+25\lambda)^2}=15$ $\Rightarrow 866\lambda^2+2598\lambda+1732=0$ $\Rightarrow \lambda^2+3\lambda+2=0$ $\Rightarrow \lambda=-1,\ -2$ $\Rightarrow$ B $(4,10,18)$, C $(0,-5,-7)$	**M1** **A1** **M1** **A1** **B1** **A1**	Finding AB in terms of λ For B For C	B1 can also be given for verifying AB = 15 and showing B is on line
			6		
	(ii)	$\text{AC}=[2,10,11]$ $\mathbf{n}=\begin{pmatrix}2\\10\\11\end{pmatrix}\times\begin{pmatrix}4\\15\\25\end{pmatrix}=\begin{pmatrix}85\\-6\\-10\end{pmatrix}$ $\Rightarrow 85x-6y-10z=c$ Sub one value to give c $\Rightarrow 85x-6y-10z=100$	**B1** **M1** **A1** **A1**	Or any vector in plane other than BC Suitable vector product or other method for finding normal	
			4		

	(iii)	This plane contains the line BC and **n** $\mathbf{n'} = \begin{pmatrix} 85 \\ -6 \\ -10 \end{pmatrix} \times \begin{pmatrix} 4 \\ 15 \\ 25 \end{pmatrix} = \begin{pmatrix} 0 \\ -2165 \\ 1299 \end{pmatrix} = \begin{pmatrix} 0 \\ -5 \\ 3 \end{pmatrix}$ $\Rightarrow 5y - 3z = c$ Sub one value to give c $\Rightarrow 5y - 3z + 4 = 0$	**M1** **A1** **M1** **A1**	Appropriate vector product **oe** Allow uncancelled vector Must be seen!	
			4		
	(iv)	$\mathbf{BC} = [-4, -15, -25]$ $\quad \mathbf{AD} = [-1, -4, -1]$ $\mathbf{BC} \times \mathbf{AD} = \begin{bmatrix} 4 & 15 & 25 \\ 1 & 4 & 1 \end{bmatrix} = [-85, 21, 1]$ $\lvert \mathbf{BC} \times \mathbf{AD} \rvert = \sqrt{85^2 + 21^2 + 1} = \sqrt{7667}$ **AC =** $[2, 10, 11]$ $\text{Distance} = \left\lvert \dfrac{(\mathbf{AC}).(\mathbf{BC} \times \mathbf{AD})}{\sqrt{7667}} \right\rvert$ $= \left\lvert \dfrac{-170 + 210 + 11}{\sqrt{7667}} \right\rvert = \dfrac{51}{\sqrt{7667}}$	**B1** **M1** **M1** **A1** **B1** **M1** **A1**	 Finding vector product Finding magnitude, pythagoras must be seen N.b. Answer given Other vectors possible Accept 0.582.....	
			7		

	(v)	A (2, 5, 4), B(4, 10, 18), C(0,-5, -7), D(1, 1, 3) $\mathbf{AB} = [2,5,14]$ $\mathbf{AC} = [-2,-10,-11]$ $\mathbf{AD} = [-1,-4,-1]$ Volume = $\left\lvert \frac{1}{6}\left(\mathbf{AB}\bullet(\mathbf{CA}\times\mathbf{DA}\right)\right\rvert$ $\mathbf{AC}\times\mathbf{AD} = \begin{bmatrix} -2 & -10 & -11 \\ -1 & -4 & -1 \end{bmatrix} = [-34,9,-2]$ $= \left\lvert \frac{1}{6}\left((2,5,14)\bullet(-34,9,-2)\right)\right\rvert = \left\lvert \frac{1}{6}\left(-68+45-28\right)\right\rvert$ $= \frac{51}{6} = \frac{17}{2}$	**M1** **A1** **A1**	Formula for volume Vector product	
			3		

2	**(i)**	$z = 3x^2 - 12xy + 2y^3 + 60$ $\frac{\partial z}{\partial x} = 6x - 12y = 0 \Rightarrow x = 2y$ $\frac{\partial z}{\partial y} = -12x + 6y^2 = 0 \Rightarrow y^2 = 2x = 4y$ $\Rightarrow y = 0$ or 4. $\Rightarrow x = 0$ or 8. $\Rightarrow z = 60$ or -4 Stationary points at $A(8, 4, -4)$, and $B(0, 0, 60)$	**M1** **A1** **M1** **B1** **A1**	For finding both partials and setting = 0 Both Solving simultaneously to get x or y Also by verification	
			5		
	(ii)(*A*)	$z = 3x^2 - 12xy + 2y^3 + 60$ Substitute $x = 8 + h,\ \ y = 4 + k$ $\Rightarrow z_p = 3(8+h)^2 - 12(8+h)(4+k) + 2(4+k)^3 + 60$ $= -4 + 3(h^2 - 4hk + 4k^2) + 12k^2 + 2k^3$ $= -4 + 3(h-2k)^2 + 2k^2(k+6)$	**M1** **M1** **A1**	For substitution and expansion For splitting $24k^2$	
			3		
	(ii)(*B*)	For all values of h and k, $(h-2k)^2 > 0$ and $2k^2(k+6) > 0$ providing $k > -6$ So $z > -4$ for all values of x and y close to A So is local minimum.	**M1** **M1** **A1**	Or for small k	
			3		

	(ii)(*C*)	When $x=0,\ z=2y^3+60$ and so either side of (0, 0, 60) the value of z will be greater or less than 60. When $y=0,\ z=3x^2+60$ and so either side of (0, 0, 60) the value of z will always be greater than 60. So B is a saddle point.	**M1** **A1** **A1** **A1**	For obtaining functions For pt of inflexion - can be by sketch For minimum - can be by sketch	
			4		
	(iii)	At (1, 1, 53), $\frac{\partial z}{\partial x}=6x-12y=-6$ $\frac{\partial z}{\partial y}=-12x+6y^2=-6$ Equation of tangent plane is: $z-53=\frac{\partial z}{\partial x}(x-1)+\frac{\partial z}{\partial y}(y-1)$ $z-53=-6x-6y+12$ $\Rightarrow 6x+6y+z=65$	**M1** **A1** **M1** **A1**	Finding values of derivatives Eqn of plane ag	Or $-6x\ -6y-z=c$
			4		
	(iv)	$\frac{\partial z}{\partial x}=-6$ and $\frac{\partial z}{\partial y}=-6$ $\Rightarrow x=2y-1$ and $y^2=2x-1$ $\Rightarrow y^2-4y+3=0$ $\Rightarrow y=1,3$ $\Rightarrow$ Coordinates of R $(5,3,9)$	**M1** **A1** **M1** **A1** **A1**	Put partial derivatives = –6 Both correct Solve simultaneously Both values	
			5		

3	(i)	$x = a\cos\theta,\ x' = -a\sin\theta,\ x'' = -a\cos\theta$ $y = b\sin\theta,\ y' = b\cos\theta,\ y'' = -b\sin\theta$	B1	derivatives	
		$r = \dfrac{\left((x')^2 + (y')^2\right)^{3/2}}{x'y'' - x''y'} = \dfrac{\left(a^2\sin^2\theta + b^2\cos^2\theta\right)^{3/2}}{-a\sin\theta.-b\sin\theta - -a\cos\theta.b\cos\theta}$	M1	Apply formula (or for κ)	
		$= \dfrac{\left(a^2\sin^2\theta + b^2\cos^2\theta\right)^{3/2}}{ab}$	M1	Set $\theta = 0$	
		At A, $\theta = 0 \Rightarrow r = \dfrac{\left(a^2\sin^2 0 + b^2\cos^2 0\right)^{3/2}}{ab}$	A1	unsimplified	
		$= \dfrac{\left(b^2\right)^{3/2}}{ab} = \dfrac{b^2}{a}$	A1	ag	
		The centre is on the *x*-axis *r* less than *a*	M1		
		So centre of curvature is at $\left(a - \dfrac{b^2}{a}, 0\right)$ i.e. $\left(\dfrac{a^2 - b^2}{a}, 0\right)$	A1		
			7		
	(ii)	Radius = $\dfrac{a^2}{b}$	B1		
		Centre is at $\left(0, b - \dfrac{a^2}{b}\right)$ i.e. $\left(0, \dfrac{b^2 - a^2}{b}\right)$	B1		
			2		
	(iii)	$\dfrac{a^2}{b} = \dfrac{b^2}{a} \Rightarrow a = b$ The ellipse is a circle OR the centre of curvature for both points (and all points) is at (0, 0)	B1		
			1		

	(iv)	$s=\int_0^{\pi/2}\sqrt{\left(\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2\right)}\,d\theta$	M1	Applying formula	
		$=\int_0^{\pi/2}\sqrt{\left((a\sin\theta)^2+(b\cos\theta)^2\right)}\,d\theta$			
		$=\int_0^{\pi/2}\sqrt{\left(a^2\sin^2\theta+b^2\cos^2\theta\right)}\,d\theta$	A1		
		$=\int_0^{\pi/2}\sqrt{\left(b^2\sin^2\theta+b^2\cos^2\theta+(a^2-b^2)\sin^2\theta\right)}\,d\theta$	M1	Eliminate $\cos\theta$	
		$=\int_0^{\pi/2}\sqrt{\left(b^2+(a^2-b^2)\sin^2\theta\right)}\,d\theta$	A1		
		$=b\int_0^{\pi/2}\sqrt{\left(1+\frac{(a^2-b^2)}{b^2}\sin^2\theta\right)}\,d\theta$			
		$\Rightarrow\lambda^2=\frac{(a^2-b^2)}{b^2}$	A1		
		When $a=b$, $\lambda=0$			
		$\Rightarrow s=b\int_0^{\pi/2}d\theta=\frac{1}{2}b\pi$	A1		
		This is a quarter of the circumference of a circle	A1	Or arc length of part of circle	
			7		

	(v)	Centre of curvature is at $(a\cos\theta - \rho\sin\psi, b\sin\theta + \rho\cos\psi)$	M1	Parametric equations	Or find equation of normal
		where $\rho = \frac{\left(a^2\sin^2\theta + b^2\cos^2\theta\right)^{3/2}}{ab}$			
		giving			
		$x = a\cos\theta - \frac{\left(a^2\sin^2\theta + b^2\cos^2\theta\right)^{3/2}}{ab}.\frac{b\cos\theta}{\left(a^2\sin^2\theta + b^2\cos^2\theta\right)^{1/2}}$	M1 A1	Deal with ψ	Or partial diffn of normal Eqn for x or y
		$\Rightarrow ax = \cos\theta\left(a^2 - \left(a^2\sin^2\theta + b^2\cos^2\theta\right)\right)$ $= \cos\theta\left(a^2 - b^2\right)\cos^2\theta = \cos^3\theta\left(a^2 - b^2\right)$	A1		
		Similarly $by = \sin^3\theta\left(b^2 - a^2\right)$	A1		
		$\Rightarrow \left(\frac{ax}{a^2 - b^2}\right)^{2/3} + \left(\frac{by}{a^2 - b^2}\right)^{2/3} = 1$ **oe**	M1 A1	i.e. $(ax)^{2/3} + (by)^{2/3} = \left(a^2 - b^2\right)^{2/3}$	
			7		

4	(i)	The set is closed. i.e. $\begin{pmatrix} a & b \\ 0 & \frac{1}{a} \end{pmatrix}\begin{pmatrix} c & d \\ 0 & \frac{1}{c} \end{pmatrix} = \begin{pmatrix} ac & ad+\frac{b}{c} \\ 0 & \frac{1}{ac} \end{pmatrix}$ Identity is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or m(1,0) Each element has an inverse Inverse of $\begin{pmatrix} a & b \\ 0 & \frac{1}{a} \end{pmatrix}$ is $\begin{pmatrix} \frac{1}{a} & -b \\ 0 & a \end{pmatrix}$ Which is in M	**M1** **A1** **A1** **B1** **M1** **A1** **A1**	Attempt to demonstrate closure Two general and different matrices Correct product therefore shows closure Attempt to demonstrate inverse A general matrix and its inverse	
			7		
	(ii)	e.g. product $\begin{pmatrix} c & d \\ 0 & \frac{1}{c} \end{pmatrix} \times \begin{pmatrix} a & b \\ 0 & \frac{1}{a} \end{pmatrix} = \begin{pmatrix} ac & cb+\frac{d}{a} \\ 0 & \frac{1}{ac} \end{pmatrix}$ OR one numeric example. e.g. $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 0 & .5 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 0 & .5 \end{pmatrix}$ but $\begin{pmatrix} 2 & 3 \\ 0 & .5 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 7 \\ 0 & .5 \end{pmatrix}$ So No	**M1** **A1** **A1**	Demonstration by multiplying 2 matrices each way round (one way might be quoted from (i)) 2 correct (and different) products Dep on previous 2 marks	
			3		

	(iii)	e.g. $\begin{pmatrix} k & b \\ 0 & \frac{1}{k} \end{pmatrix}\begin{pmatrix} k & c \\ 0 & \frac{1}{k} \end{pmatrix} = \begin{pmatrix} k^2 & ..... \\ 0 & \frac{1}{k^2} \end{pmatrix}$ This is only in the set N_k if $k^2 = k$ Given $k \neq 0 \Rightarrow k = 1$ only	**M1** **A1** **A1** **A1**	Multiplying 2 matrices in N (Allow matrices the same) Sight of k^2	
			4		
	(iv)	$m(1,1)^2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$	**M1** **A1**		
			2		
	(v)	$\begin{matrix} & m(1,0) & m(1,1) & m(1,2) & m(1,3) \\ m(1,0) & m(1,0) & m(1,1) & m(1,2) & m(1,3) \\ m(1,1) & m(1,1) & m(1,2) & m(1,3) & m(1,0) \\ m(1,2) & m(1,2) & m(1,3) & m(1,0) & m(1,1) \\ m(1,3) & m(1,3) & m(1,0) & m(1,1) & m(1,2) \end{matrix}$	**B1** **B3**	m(1,0) the identity –1 each error	
			4		
	(vi)	Group table for R OR Any argument that states that: *a*, *b*, *c* have order 2 **B1** reason for this **B1** but only one element of P has order 2 **B1** So No	**B3** **B1**	–1 each error Dependent on previous B3	
			4		

5	**(i)**	$\mathbf{P} = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 0 & 1/3 & 0 \\ 1/2 & 1/3 & 1/2 \\ 1/2 & 1/3 & 1/2 \end{pmatrix} \end{array}$	**B2**	B1 for two out of three columns correct	
			2		
	(ii)(*A*)	$\mathbf{P}^3 p$ $\begin{pmatrix} 0.138\dot{8} & - & - \\ - & - & - \\ - & - & - \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ gives $0.138\dot{8}\left(=\frac{5}{36}\right)$	**M1** **A1** **A1**	Cube P Sight of matrix soi	Allow M1 for P^4
			3		
	(ii)(*B*)	$P = M^5 p$ $\mathbf{P}^5 = \begin{pmatrix} \ldots & - & - \\ - & 0.4285 & - \\ - & - & 0.4286 \end{pmatrix}$ $p = 0.5 \times 0.4285 + 0.5 \times 0.4286$ $= 0.4286$	**M1** **M1** **A1** **A1**	Using diagonal elements from P^5 Using probabilities from 2nd day Ft Cao	
			4		
	(iii)	0.143, 0.429, 0.429 $\left(=\frac{1}{7}, \frac{3}{7}, \frac{3}{7}\right)$ Over a long period these are the probabilities that on any day at random the inspector is at these factories	**M1** **B1** **A1** **B1**	Obtaining equations Sight of $x + y + z = 1$ soi	Or **M1** considering P^n where n is large.(10 or more) **A2** for probabilities, A1 one error
			4		

	(iv)	$\mathbf{Q} = \begin{matrix} & A & B & C \\ A & 0.8 & 1/3 & 0 \\ B & 0.1 & 1/3 & 1/2 \\ C & 0.1 & 1/3 & 1/2 \end{matrix}$	**B1**		
			M1	Solve or consider Q^n for large n.	
		0.455, 0.273, 0.273 $\left(= \frac{5}{11}, \frac{3}{11}, \frac{3}{11}\right)$	**A1**		
			3		
	(v)	P(from A to A) = 0.8. So $\alpha = 0.8$	**B1**		
		$\Rightarrow \frac{\alpha}{1-\alpha} = \frac{0.8}{0.2} = 4$	**M1** **A1**	For using $\frac{\alpha}{1-\alpha}$ or $\frac{1}{1-\alpha}$	
			3		
	(vi)	New transition matrix: $\mathbf{R} = \begin{matrix} & A & B & C \\ A & 1 & 1/3 & 0 \\ B & 0 & 1/3 & 1/2 \\ C & 0 & 1/3 & 1/2 \end{matrix}$	**M1**	Method by "trial" with new matrix.	
		We need 3rd entry of 2nd row to be < 0.1 $\mathbf{R}^9 = \begin{pmatrix} ... & ... & ... \\ ... & ... & 0.1162 \\ ... & ... & ... \end{pmatrix}$, $\mathbf{R}^{10} = \begin{pmatrix} ... & ... & ... \\ ... & ... & 0.0969 \\ ... & ... & ... \end{pmatrix}$	**A1**	For sight of one value	
		So 10 days later (which is day 25).	**A1**		
			3		
	(vii)	A is the absorbing state.	**B1**		
		If it goes to A then it stays there.	**B1**	**oe**	
			2		

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4757 Further Applications of Advanced Mathematics (FP3)

General Comments

Most candidates for this paper were well-prepared, and were able to produce substantial attempts at all three of their chosen questions. Q.1 (on vectors) and Q.2 (on multi-variable calculus) were the most popular questions, each chosen by about 80% of the candidates. By far the least popular question was Q.3 (on differential geometry), which was chosen by about one third of the candidates. The standard of presentation, and the accuracy of algebraic and numerical work, was generally good throughout the paper.

Comments on Individual Questions

Q.1(i) Many candidates appeared to be unsure of how to tackle this; for example, starting by finding the perpendicular distance from A to the line, followed by some unproductive work, before settling down to the correct method. This consisted of expressing the distance from A to a general point on the line in terms of λ, and hence forming an equation. When this was done, the equation was almost always solved correctly to find the points B and C. Some candidates produced the correct points B and C with no apparent reasoning (presumably by trial); this was acceptable provided that *both* the required conditions (lying on the given line, and distance 15 from A) were convincingly verified.

Q.1(ii) The usual method was to use a vector product to find a normal vector to the plane; this was almost always completed successfully. Those who started with a parametric equation for the plane then eliminated the parameters, or who started with three simultaneous equations, were much more likely to make careless errors.

Q.1(iii) A substantial number of candidates scored no marks in this part. The required normal vector is perpendicular to the line BC and to the normal found in part (ii). Candidates who realised this could form a suitable vector product, which was then usually evaluated accurately. It was acceptable to take the given answer and verify that it satisfied the required conditions; but a candidate would not be given any credit unless *all* the conditions (containing BC and being perpendicular to the plane ABC) were considered. An elegant method, used by a few candidates, was to spot that the required normal vector is AM, where M is the mid-point of BC.

Q.1(iv) The method for finding the shortest distance between skew lines was very well-understood, and most candidates answered this part confidently. Several candidates lost marks because they just stated that the length of the vector (−85, 21, 1) was $\sqrt{(7667)}$ without showing how this was calculated. As always, when an answer is given on the question paper, full and convincing working must be shown.

Q.1(v) The method for finding the volume of a tetrahedron, using a scalar triple product, was well known, and the correct answer was very often obtained.

Q.2(i) The partial differentiation was usually done correctly, and the stationary points found successfully.

Q.2(ii)(*A*) This involved substituting into the equation of the surface and rearranging. Many candidates lost marks because they did not show convincingly how the required result followed from their working. Some candidates tried to use an approximate result for small changes using the partial derivatives, although an exact result was required here.

Q.2(ii)(*B*) This was clearly unfamiliar work, and most candidates scored no marks in this part. Many candidates thought that both second derivatives being positive (or the sections in $x = 8$ and in $y = 4$ both having a minimum) was sufficient to show that the surface had a minimum. To obtain marks, it was necessary to explain convincingly how the result in part (*A*) shows that $p > -4$ for all small values of h and k.

Q.2(ii)(*C*) In this part, most candidates found the required sections and correctly deduced that B was a saddle point.

Q.2(iii) This part was very well-answered.

Q.2(iv) Candidates who started with $\partial z/\partial x = \partial z/\partial y = -6$ were usually able to complete this successfully. Having obtained $y = 3$, $x = 5$, $z = 9$, a fair number of candidates gave the coordinates of R as (3, 5, 9) instead of (5, 3, 9). Common misunderstandings were $\partial z/\partial x = \partial z/\partial y = 6$ and $\partial z/\partial x = \partial z/\partial y = 6\lambda$.

Q.3(i) Most candidates were able to use the parametric version of the formula to find the radius of curvature. Some tried to use the version involving dy/dx and d^2y/dx^2, but d^2y/dx^2 was never found correctly. The centre of curvature was often omitted, or incorrect. Many worked out the normal vector at A by differentiation rather than by inspection from the diagram, and some went the wrong way.

Q.3(ii)-(iv) These parts were generally answered well.

Q.3(v) Most candidates approached this by finding the equation of the normal to the ellipse at a general point, then differentiating partially with respect to θ. Others found the centre of curvature at a general point by using the radius of curvature and the unit normal vector, but this was more complicated. It was then necessary to obtain x and y in their simplest forms so that θ could be eliminated to obtain the cartesian equation. This provided a considerable challenge, and few candidates were able to complete it.

Q.4(i) Most candidates dealt with the identity and inverses correctly. For closure, very many candidates thought that it was sufficient to consider the square of a general element.

Q.4(ii) Most candidates were able to demonstrate that the group was not commutative Some confused commutativity with associativity.

Q.4(iii) Those who considered $k^2 = k$ as the condition for closure very often scored full marks. Some candidates gave a fully correct proof based on $kc + b/k$ having to be an integer for all integers b and c.

Q.4(iv)-(v) These parts were answered correctly by almost every candidate.

Q.4(vi) Most candidates correctly decided that R was not isomorphic to P, although this assertion was not always fully justified. Those who wrote out the combination table for R could easily show that the patterns were different. Many candidates based their arguments on the orders of the elements; and here, if the table for R was not given, they were also required to explain why the common value of $a*a = b*b = c*c$ had to be e. The simplest argument, given by quite a few candidates, was 'In P, the squares of the three non-identity elements are not all equal, but in R, $a*a = b*b = c*c$.'

Q.5(i)-(ii)(*A*) These parts were almost always answered correctly.

Q.5(ii)(*B*) Most candidates appreciated that the factories visited on Day 2 and Day 7 were not independent, and so they needed to use the probabilities for Day 2 and the diagonal elements of $\mathbf{P}^5$ to calculate the required probability.

Q.5(iii) Most candidates obtained the correct equilibrium probabilities and explained that these were the long run probabilities of visiting each factory. Some solved simultaneous equations resulting from the equilibrium condition, but the most common method was to consider $\mathbf{P}^n$ for large values of n. Having obtained the limiting matrix $\mathbf{P}^n$, some candidates did not write out the resulting probabilities for the factories A, B, C.

Q.5(iv) This part was very well answered.

Q.5(v) Many candidates used the formula $p / (1 – p)$ or $1 / (1 – p)$ to find the expected run length, usually with the correct value $p = 0.8$. Then 1 might be added or subtracted to give their answer. The correct answer (4) very often resulted, but 5 was a very common error.

Q.5(vi) The transition matrix **R** was usually given correctly, and many candidates considered the appropriate element in the powers $\mathbf{R}^n$. Answers of Day 24 or Day 26 (instead of Day 25) were fairly common.

Q.5(vii) Most candidates understood the concept of an absorbing state.

Oxford Cambridge and RSA

GCE Mathematics (MEI)											
				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	48	43	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	56	50	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	56	51	46	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	74	67	60	54	48	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	62	57	53	49	45	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	59	52	46	40	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	38	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	62	54	46	39	32	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	54	47	40	33	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	65	60	55	50	46	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	64	58	52	47	42	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	56	51	46	41	37	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	54	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	45	40	34	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
			UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	54	47	41	35	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	54	47	41	35	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	56	51	46	41	37	0
		UMS	100	80	70	60	50	40	0